(8 pages)

Reg. No. : ....

Code No. : 20381 E **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The n^{th} degree equation $f(x)=0$ cannot have more than ———— roots

   (a) 4 (b) 6

   (c) 7 (d) n

2. If α, β, γ are the roots of the equation $x^4+px^3+qx^2+rx+5=0$ then $\sum \alpha\beta\gamma =$ ————

   (a) $-p$ (b) q

   (c) $-r$ (d) s

3. After removing the fractional coefficients from the equation $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$ we get ————

   (a) $x^3-1=0$

   (b) $12x^3-3x^2+4x-12=0$

   (c) $x^3-3x^2+48x-1728=0$

   (d) $x^3-3x^2+48x-1=0$

4. How many imaginary roots will occur for the equation $x^7-3x^4+2x^3-1=0$?

   (a) atmost four

   (b) exactly four

   (c) atleast four

   (d) none of these

5. The characteristic equation of $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ is ——————

(a) $\lambda^2 - 2\lambda - 1 = 0$

(b) $\lambda^2 + 2\lambda - 1 = 0$

(c) $\lambda^2 - 2\lambda + 1 = 0$

(d) $\lambda^2 + 2\lambda + 1 = 0$

6. Two eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are –2 and 3, the third eigen value is ——————.

(a) 4 (b) 5

(c) 6 (d) –1

7. The Clairauts equation is ——————

(a) $y = cx + f(c)$

(b) $y = px + f(p)$

(c) $\frac{dy}{dx} = \left\{p + x\frac{dp}{dx}\right\} + f'(p)\frac{dp}{dx}$

(d) none of these

8. The partial differential equation obtained from $Z = ax + by + a^2$ by eliminating the arbitrary constants 'a' and 'b' is ——————

(a) $Z = px + py + a^2$ (b) $Z = qx + py + a^2$

(c) $Z = px + qy + a^2$ (d) none of these

9. $L(x) =$ ——————

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $-\frac{1}{s^2}$ (d) none of these

10. $L^{-1}\left[\frac{1}{s-a}\right] =$ ——————

(a) 1 (b) x

(c) e^{ax} (d) e^{-ax}

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that one of its root is $-2 = i\sqrt{7}$.

Or

(b) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in Harmonic progression.

12. (a) Diminish the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and hence solve the original equation.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the eigen value and eigen vectors of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

Or

(b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$.

14. (a) Form the partial differential equation by eliminate arbitrary constants 'a' and 'b' from $\log(az - 1) = x + ay + b$.

Or

(b) Form a partial differential equation by eliminating arbitrary functions 'ϕ' from $\phi\left(x + y + z, x^2 + y^2 - z^2\right) = 0$.

15. (a) Find $L(\sin 2t \sin 3t)$.

Or

(b) (i) Prové that $L\left[e^{-ax}\right] = \frac{1}{s+a}$

(ii) If $L[f(x)] = F(s)$ then prove that $L[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in arithmetic progression if $2q^3 + 27p^2s = 9pqr$.

Or

(b) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

17. (a) Find by Horner's method, the positive root of $x^3 - 3x + 1 - 0$ lies between 1 and 2, Calculate it to three place of decimals.

Or

(b) Obtain by Newtons method, the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

18. (a) Find the eigen value and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

Or

(b) Verify Cayley–Hamilton theorem for $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

19. (a) Solve $x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y)$.

Or

(b) Solve $xp^2 - 2py + x = 0$.

20. (a) Find $L^{-1}\left[\dfrac{s^2 - s + 2}{s(s-3)(s+2)}\right]$.

Or

(b) Find $L^{-1}\left[\dfrac{cs + d}{(s+a)^2 + b^2}\right]$.
